

**Mathematics Oral Session 2 Titles and Abstracts**  
**Oral Session 2:** 9:15 – 10:45 am Saturday, November 7<sup>th</sup>, 2009

**O2-1**

**Presenter:** Melati Nungsari, Knox College

**Title:** Computing Polynomial Invariants for Knots

**Advisor:** Pedro Teixeira, Mathematics Department, Knox College

We give a brief introduction to knots and links and then delve into polynomial invariants, discussing their uses in classifying knots and detecting chirality. We then present an algorithm used in calculating polynomial invariants, and an implementation of this algorithm in Mathematica that takes a Gauss Code representing a knot or link and returns its HOMFLY, Jones, Alexander, and Conway polynomials.

**O2-2**

**Presenter:** Melissa Yeung, The University of Chicago

**Title:** Symmetric Saddle Towers

**Advisor:** Michael Dorff, Department of Mathematics, Brigham Young University

A surface is minimal if its mean curvature vanishes identically. Minimal surfaces can be modeled by soap films spanning a wire frame and have practical applications in physics, chemistry, and architecture. We can parameterize a minimal surface using the Weierstrass Representation, and from this, determine the geometry of the surface. In this presentation, we will use techniques from harmonic and univalent function theory to offer a new proof that the projection of the symmetric saddle towers, a higher-order generalization of Scherk's singly periodic surface, is univalent. As a consequence, it follows that the symmetric saddle towers are embedded.

**O2-3**

**Presenter:** Andrei Tarfulea, The University of Chicago

**Title:** Alternating Nested Traps Characterize Parity Games

**Advisor:** Dr. Sasha Rubin, Department of Mathematics, Cornell University

In Parity Games, two players called EVEN and ODD compete by restricting each other's movements in an infinite play on a finite graph. Parity games gained major attention from the Model Checking problem in Modal  $\mu$ -Calculus and are of particular significance in Computational Complexity; they remain one of the few natural problems known to be in  $NP \setminus co-NP$  but not known to be in  $P$ . To every parity game we associate a natural number called its Trap-Depth. Via purely structural means, the trap-depth and its construction embody the techniques each player may use to restrict the other's options. This number, bounded by the size of the graph, characterizes games on which a given player has non-empty winning region. Moreover, we supply an algorithm that runs in polynomial time which solves parity games of trap-depth 1. Future work may extend this to a polynomial time algorithm for parity games of arbitrary trap-depth.

**O2-4**

**Presenters:** Augustus Lidaka and Shitanshu Aggarwal, Grinnell College

**Title:** GPU Parallelization of a Maximum Entropy Learning Model

**Advisor:** Jerod J. Weinman, Department of Computer Science, Grinnell College

Text recognition models require enormous amounts of data to train effective classifiers. The time required to train on such large data sets makes timely error analysis impractical. Many steps of the training process do not depend on one another and can be done concurrently. Such a task can be significantly accelerated

using graphics processing hardware, which contains many processing cores operating in parallel. We redesigned the training algorithm to fit a parallel architecture, Nvidia's CUDA. With various parameters and datasets, we have measured speedup in training time between a factor of 2 and 10.

## O2-5

**Presenter:** Blair R. Williams, Hope College

**Title:** Design, Implementation, and Control of an Autonomous Self-Stabilizing Bicycle

**Advisor:** Dr. Miguel Abrahantes, Hope College Department of Engineering

Riding a bicycle is a simple act that most learn at a young age, but programming a computer to do so is much more complex. However, modern control theory has enabled researchers to develop control algorithms that stabilize computer-driven bicycles to be used as military scout vehicles or for safer forms of two-wheeled transportation. Using Newtonian mechanics, we derive a simplified second-order model of a moving bicycle for feedback control analysis. Selecting the bicycle's steering angle as the control system actuating parameter, centrifugal forces (inertia) are used to manipulate the roll angle and roll angular velocity to prevent the bike from falling. To implement such a control method, a Schwinn road bike is equipped with sensors, motors, control boards, and a laptop with data acquisition capabilities. Necessary mounting affixtures are also designed and fabricated for complete onboard operation. We identify the parameters of the dynamic model using a least mean squares input-output fitting method. Based on the derived system model and identified parameters, we then implement roll angle control methods and examine system response.

This material is based on work supported by Howard Hughes Medical Institute, the Michigan Space Grant Consortium, and the Hope College Engineering Department. Bicycle parts provided by Cross Country Cycle of Holland, Michigan.

## O2-6

**Presenter:** Jeremy Bancroft Brown, The University of Chicago

**Title:** Optimization of a fuzzy C-means approach to determining probability of lesion malignancy in breast DCE-MRI

**Advisor:** Maryellen Giger, Department of Radiology, University of Chicago

**Coauthors:** Giger, Maryellen; Bhooshan, Neha; Newstead, Gillian; Jansen, Sanaz

Previous research has shown that a fuzzy C-means (FCM) approach to computerized lesion analysis has the potential to aid radiologists in the interpretation of dynamic contrast-enhanced MRI (DCE-MRI) breast exams [1][2]. Our purpose in this study was to optimize the performance of the FCM approach with respect to binary (benign/malignant) breast lesion classification in DCE-MRI. We used both raw (calculated from kinetic data points) and empirically fitted [3] kinetic features for this study. FCM was used to automatically select a *characteristic kinetic curve* (CKC) based on intensity-time point data of voxels within each lesion, using four different kinetic criteria: (1) maximum *initial enhancement*, (2) minimum *shape index*, (3) maximum *washout*, and (4) minimum *time to peak*. We extracted kinetic features from these CKC's, which were merged using linear discriminant analysis (LDA), and evaluated with receiver operating characteristic (ROC) analysis. There was comparable performance for methods 1, 2, and 4, while method 3 was inferior. Next, we modified use of the FCM method by calculating a feature vector for every voxel in each lesion and using FCM to select a *characteristic feature vector* (CFV) for each lesion. Using this method, we achieved slightly better (although  $p > 0.05$ ) performance, compared with the four CKC methods. Additionally, we proposed a method for optimizing the number of FCM clusters based upon the spatial variance of kinetic features in each lesion. Finally, we generated lesion color maps using FCM membership matrices, which facilitated the visualization of malignant voxels in a given lesion.